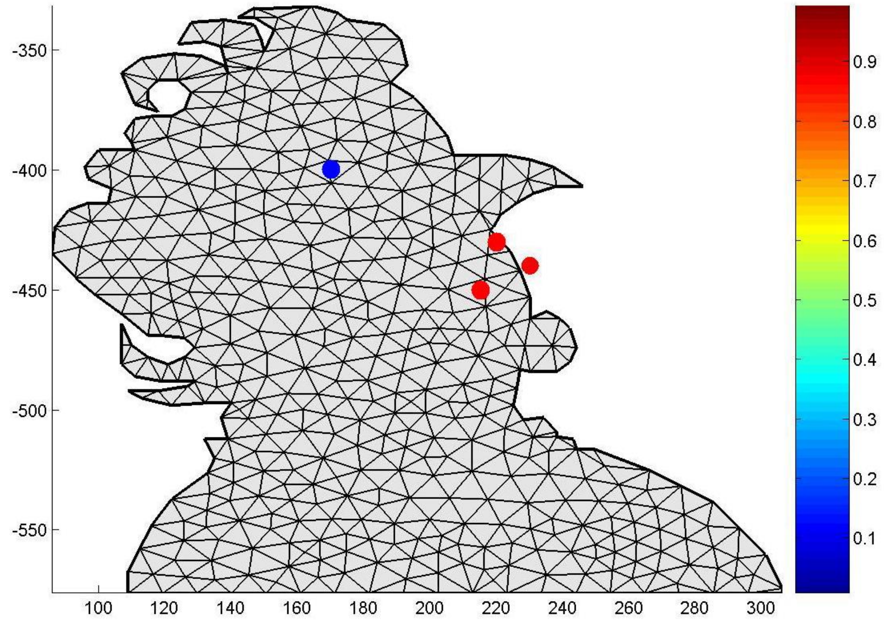
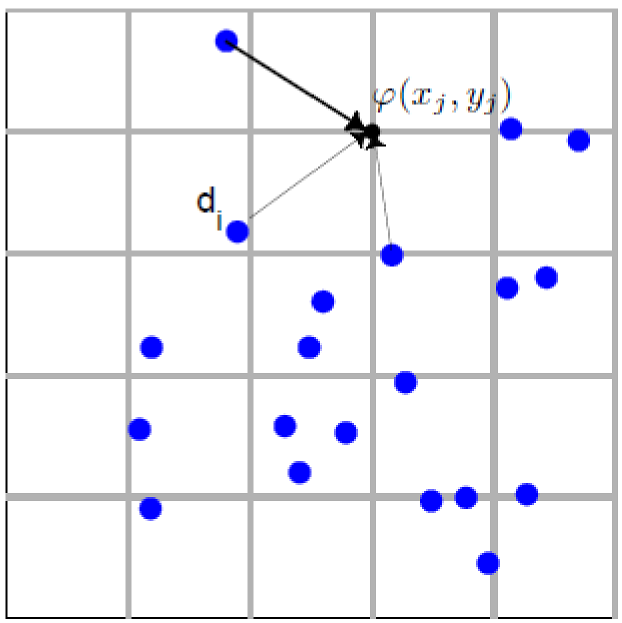
Data interpolating variational analysis for the generation of atmospheric pollution maps at various scales

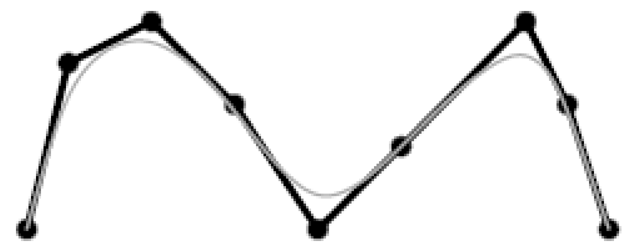
by Fabian Lenartz [1], Charles Troupin [2], Wouter Lefebvre [3]



[1] ISSeP, rue du Chéra 200, 4000 Liège, Belgium
[2] SOCIB, Parc Bit, 07121 Palma de Mallorca, Spain
[3] VITO, Boeretang 200, 2400 Mol, Belgium

Method:
Interpolation vs approximation





Refs: Gandin (1965), Bretherton et al. (1976)

Method:
Mathematical formulation and implementation

$$\varphi(\boldsymbol{r}) = \varphi_b(\boldsymbol{r}) + \varphi'(\boldsymbol{r}) = \varphi_b(\boldsymbol{r}) + \sum_{j=1}^{N_d} w_j \, d_j$$

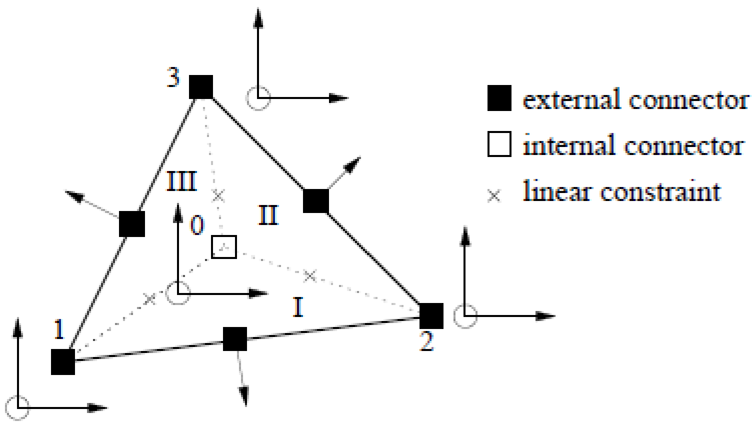
Method:
Mathematical formulation and implementation

$$J[\varphi] = \sum_{j=1}^{N_d} \mu\left[d_j - \varphi\left(x_j, y_j\right)\right]^2 + \|\varphi\|^2$$

with $\|\varphi\| = \int_D (\alpha_2 \nabla\nabla\varphi : \nabla\nabla\varphi + \alpha_1 \nabla\varphi . \nabla\varphi + \alpha_0 \varphi^2)\, dD$

Refs: Brasseur et al. (1996), Rixen et al. (2000)

Method:
Mathematical formulation and implementation



Ref.: Brasseur (1994)

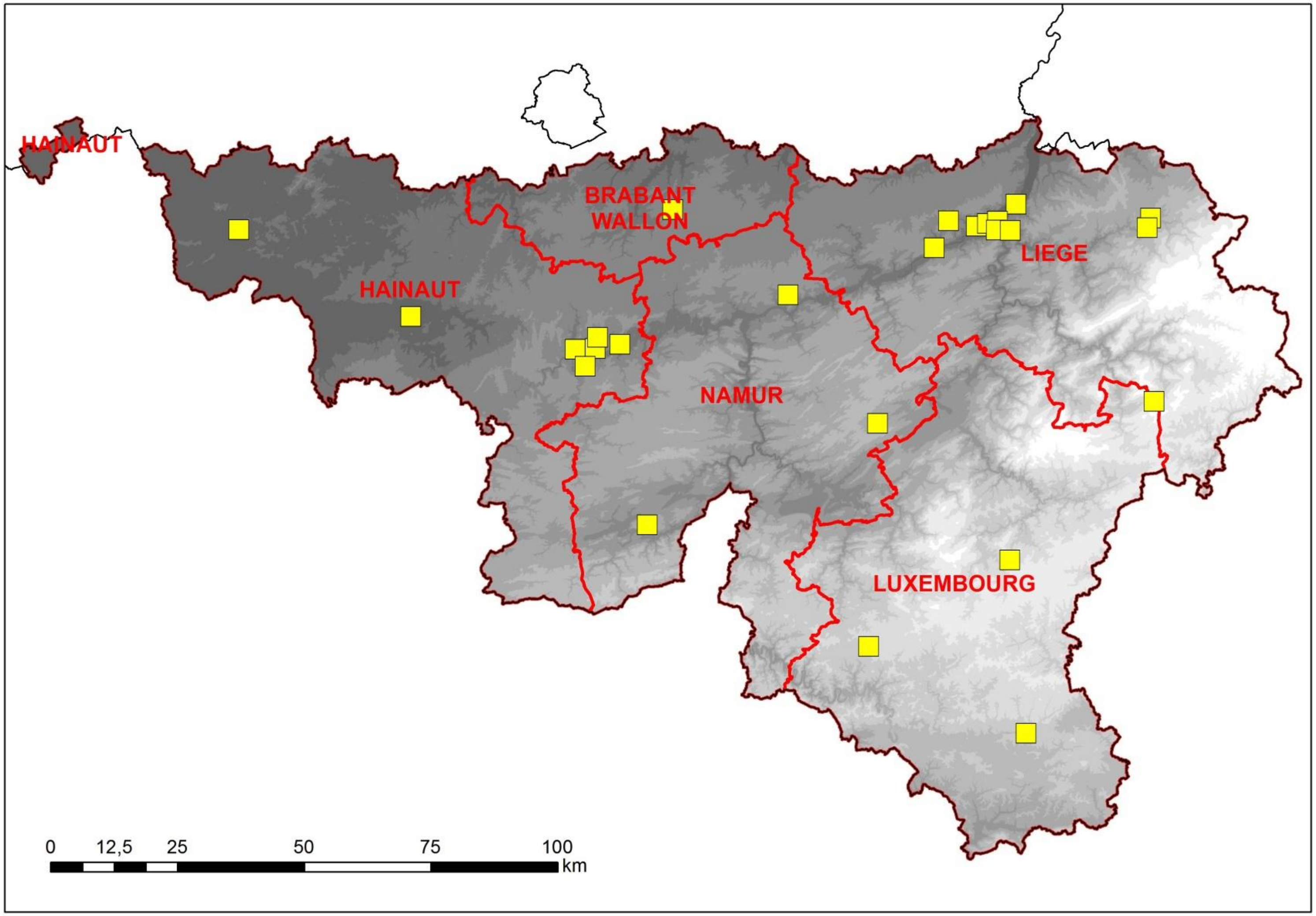
Method:
Comparison between techniques

	OI	VIM
Minimization	$e^2(\mathbf{r}) = \overline{[\varphi(\mathbf{r}) - \varphi_t(\mathbf{r})]^2}$	$J[\varphi] = \sum_{i=1}^{N_d} \mu_i [d_i - \phi(\mathbf{r_i})]^2 + \|\varphi\|^2$
Solution	$\varphi(\mathbf{r}) = \mathbf{c}^\mathsf{T}(\mathbf{r})\mathbf{D}^{-1}\mathbf{d}$	$\varphi(\mathbf{r}) = \mathbf{c}^\mathsf{T}(\mathbf{r})\mathbf{D}^{-1}\mathbf{d}$
Data correlation	$[\mathbf{D}]_{ij} = \sigma^2 c(\mathbf{r_i}, \mathbf{r_j}) + \epsilon^2 \delta_{ij}$	$[\mathbf{D}]_{ij} = K(\mathbf{r_i}, \mathbf{r_j}) + (1/\lambda)\delta_{ij}$
Data-field covariance	$[\mathbf{c}]_i = \sigma^2 c(\mathbf{r}, \mathbf{r_i})$	$[\mathbf{c}]_i = K(\mathbf{r}, \mathbf{r_i})$

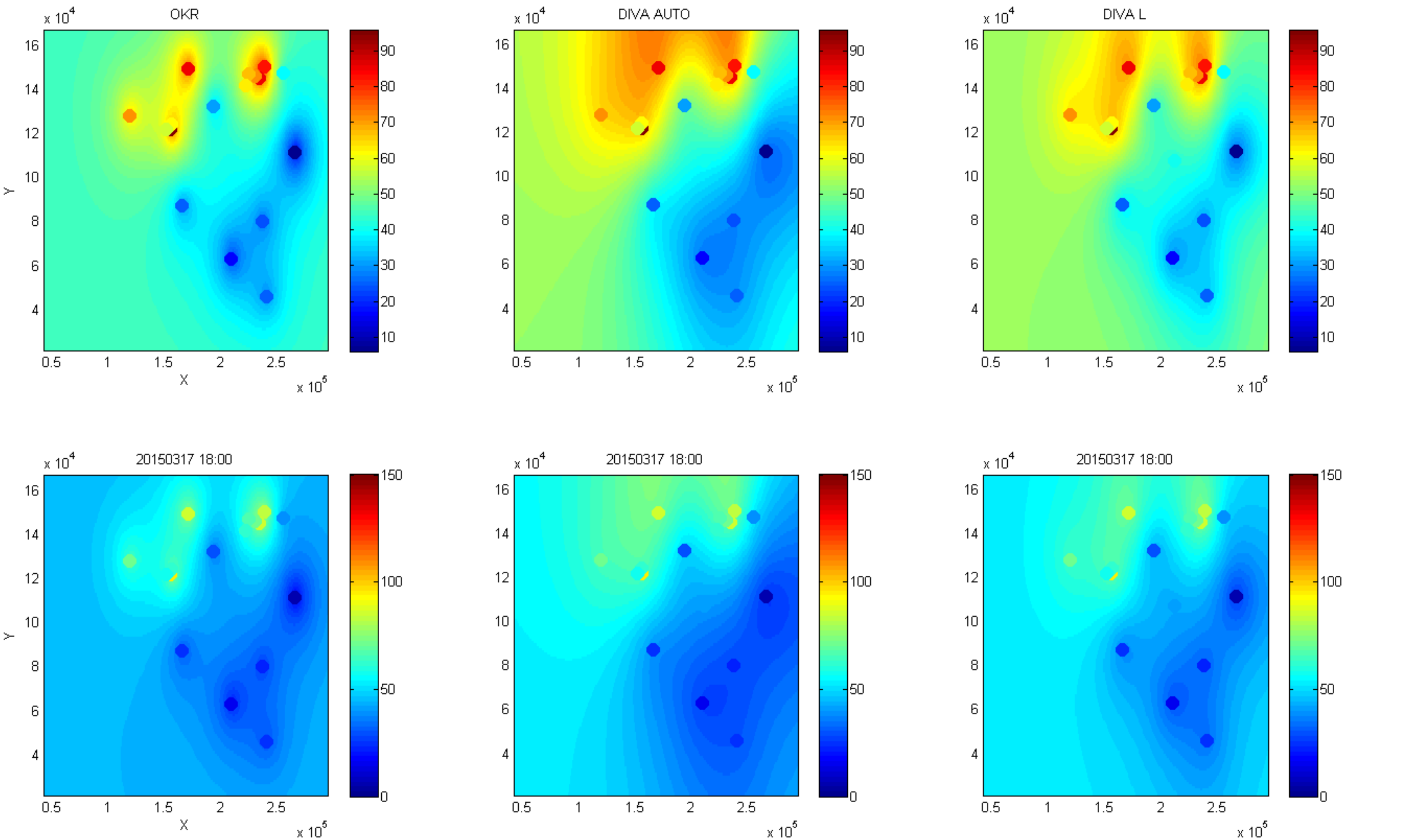
Method	$\min(\epsilon^2)$	3-D	Multivar	Ops/anal.	$\epsilon(\mathbf{r})$	*a priori*	C.V.	anis.
Cressman		•	•	$N_d N_a$		$w(r/L)$	(L)	(•)
OI	•	•	•	$N_d^3 + N_d N_a$	•	$c(r/L)$	$L, \sigma^2/\epsilon^2$	(•)
VIM	•	(•)	(•)	$N_a^{5/2}$	•	$K(r/L)$	$L, \sigma^2/\epsilon^2$	(•)

Ref.: Troupin et al. (2013)

Application – Test case N.1: Wallonia

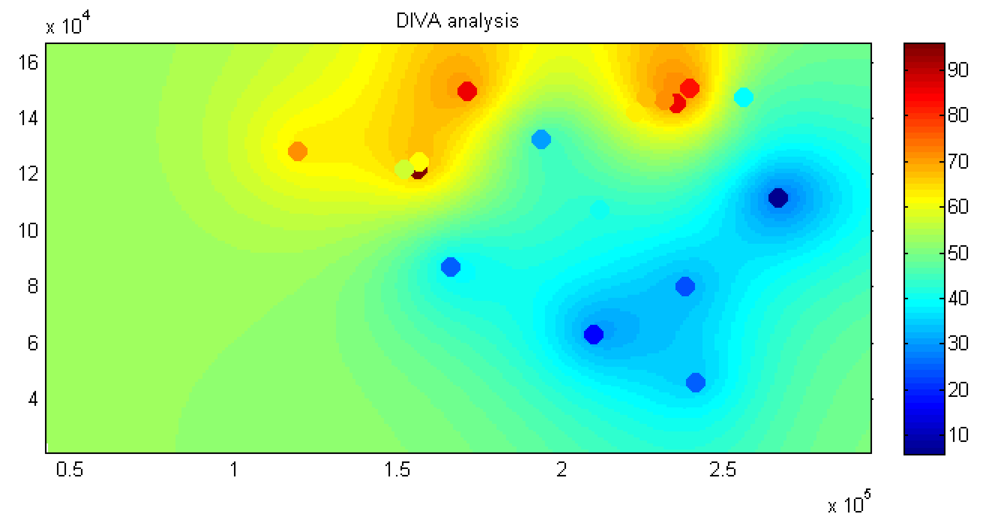


Application – Test case N.1: Wallonia



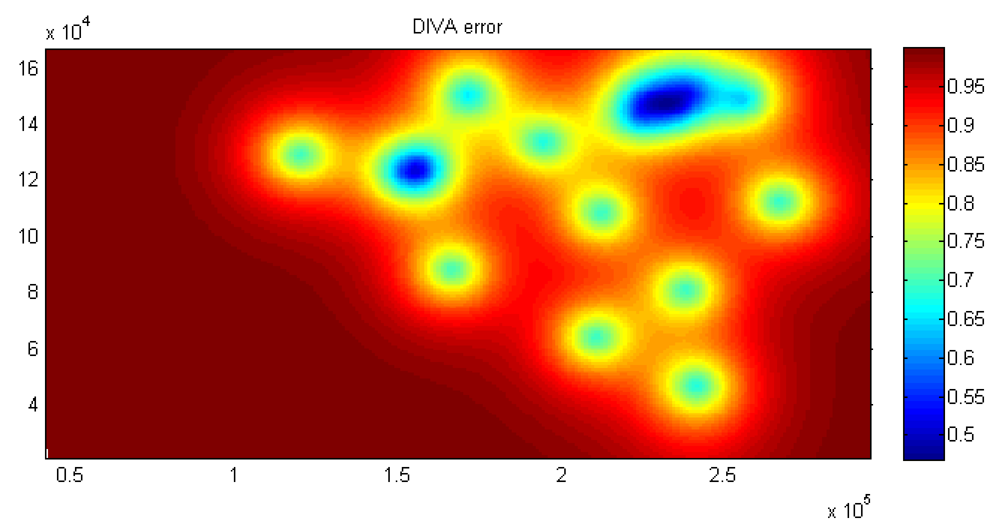
Application – Test case N.1: Wallonia

Analysis (µg/m³)



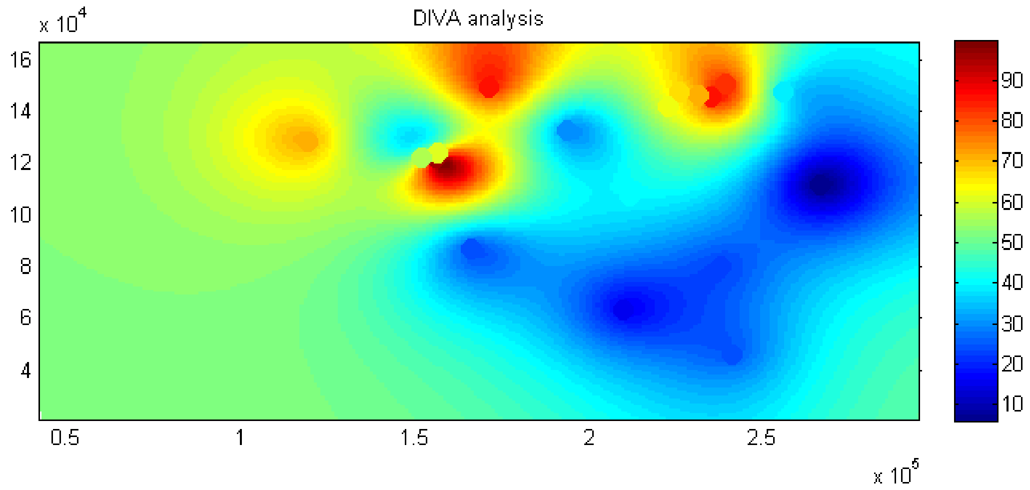
Basic test case

Error (%)

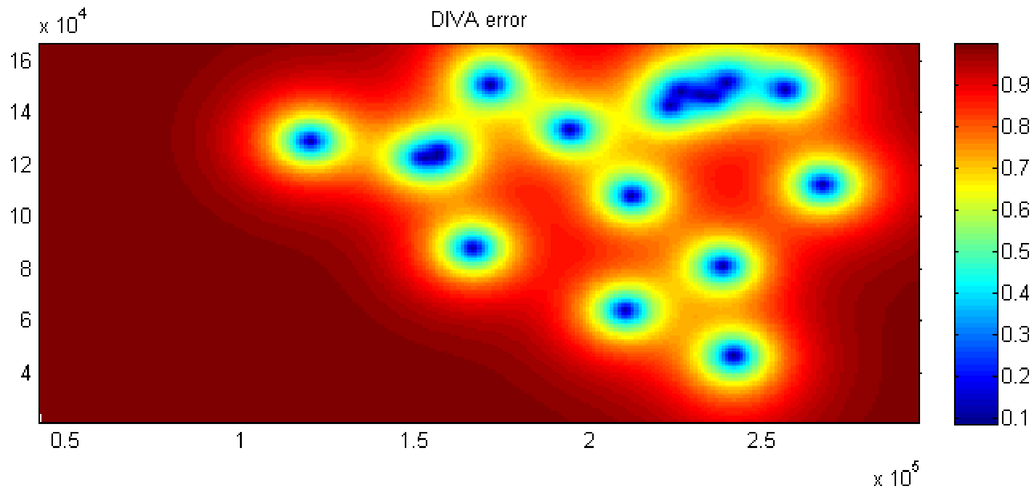


Application – Test case N.1: Wallonia

Analysis
(µg/m³)



Error
(%)



Impact of SNR modification

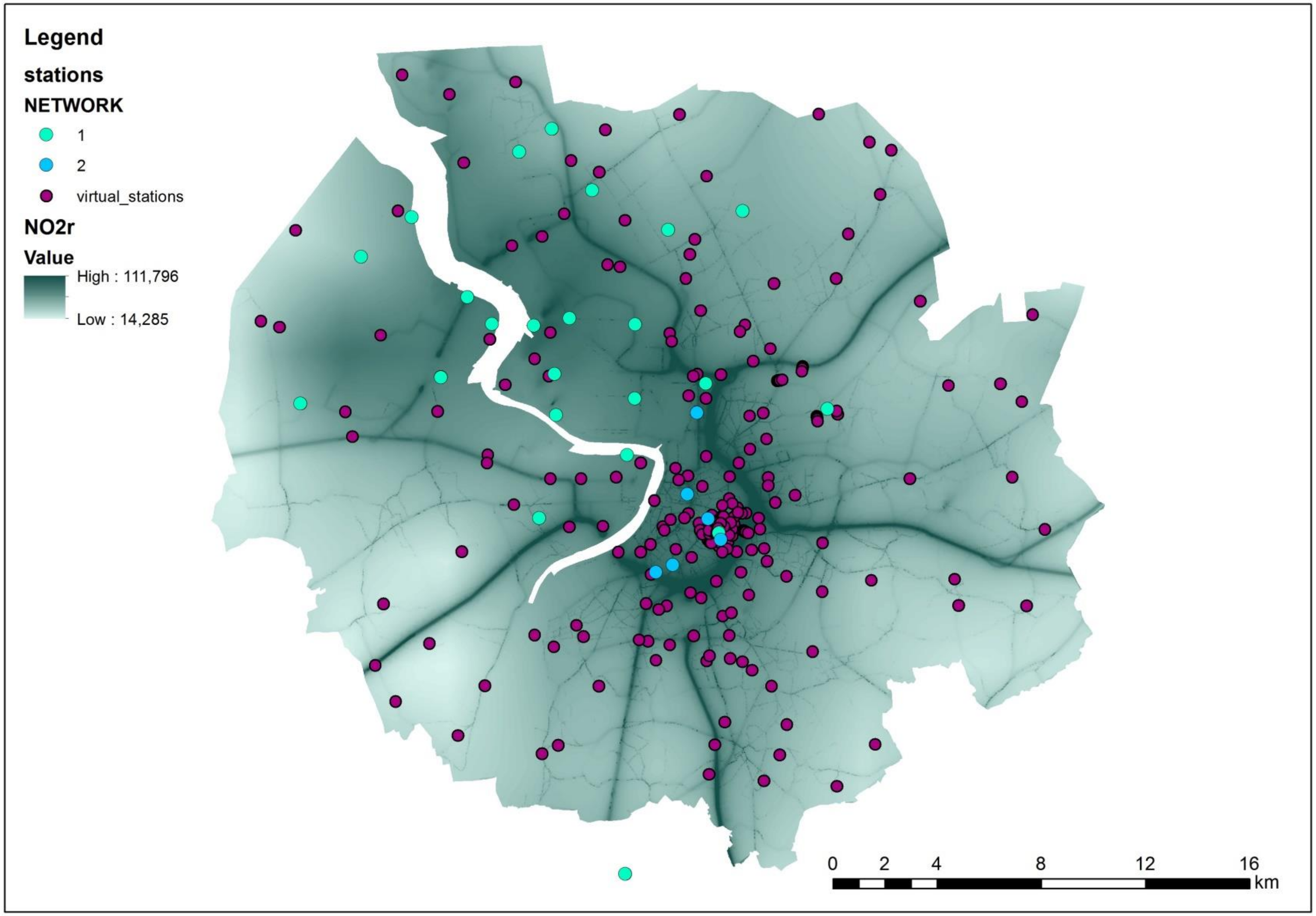
Application – Test case N.1: Wallonia

$$\tilde{J}[\varphi] = J[\varphi] + \frac{\theta}{U^2 L^2} \int_{\widetilde{D}} [\boldsymbol{u}.\widetilde{\boldsymbol{\nabla}}\varphi - \frac{A}{L}\widetilde{\boldsymbol{\nabla}}.\widetilde{\boldsymbol{\nabla}}\varphi]^2 d\widetilde{D}$$

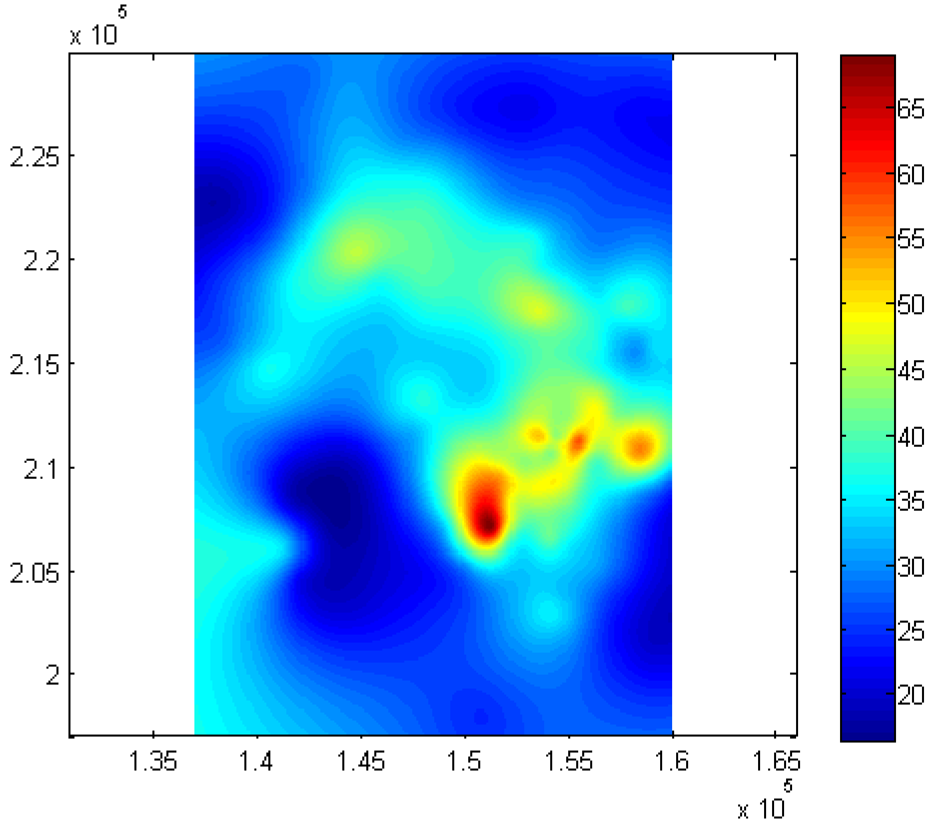
Physical constraint:
Advection - Diffusion

Application – Test case N.2: Antwerp

Ref.: Lefebvre et al. (2016)

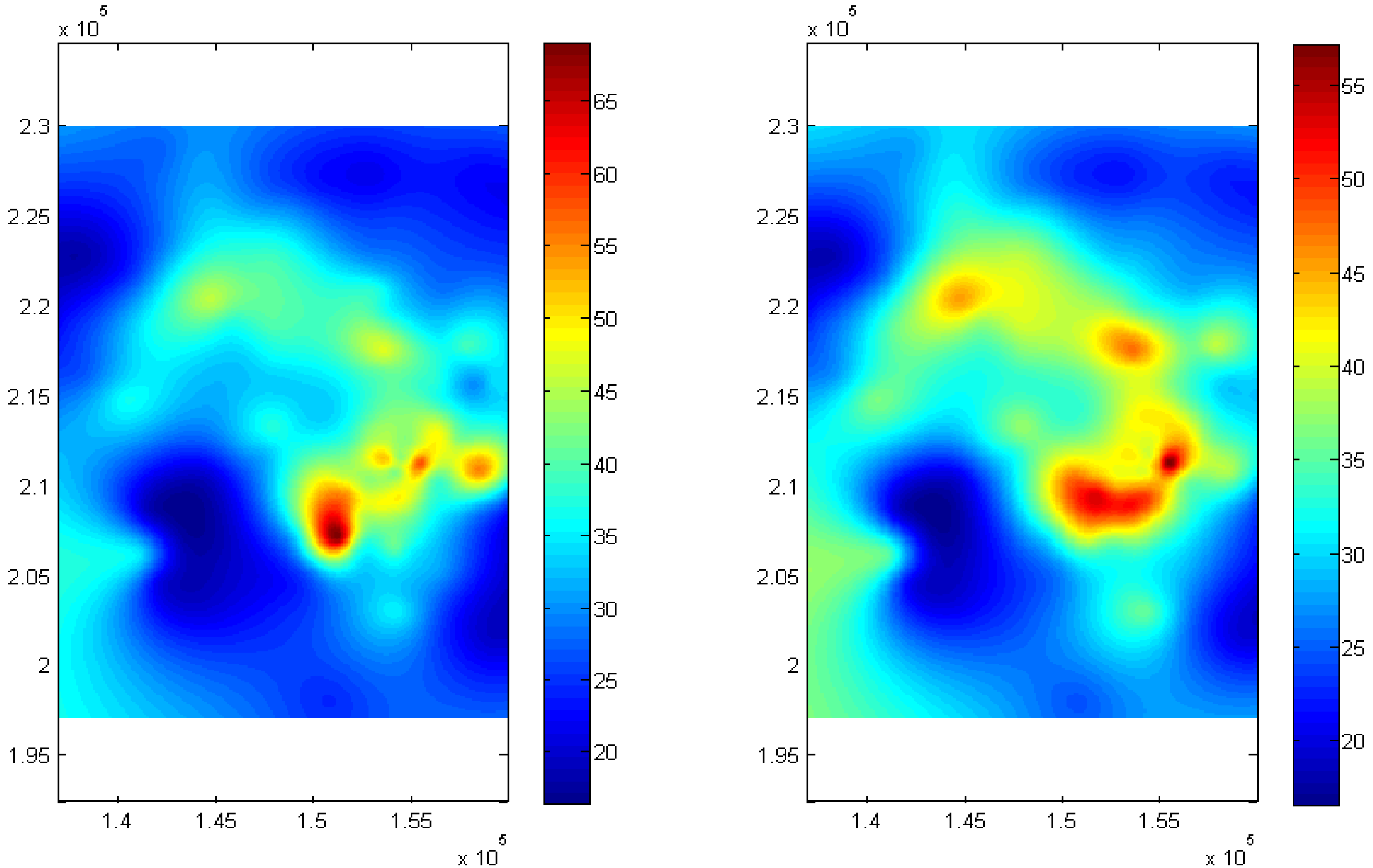


Application – Test case N.2: Antwerp



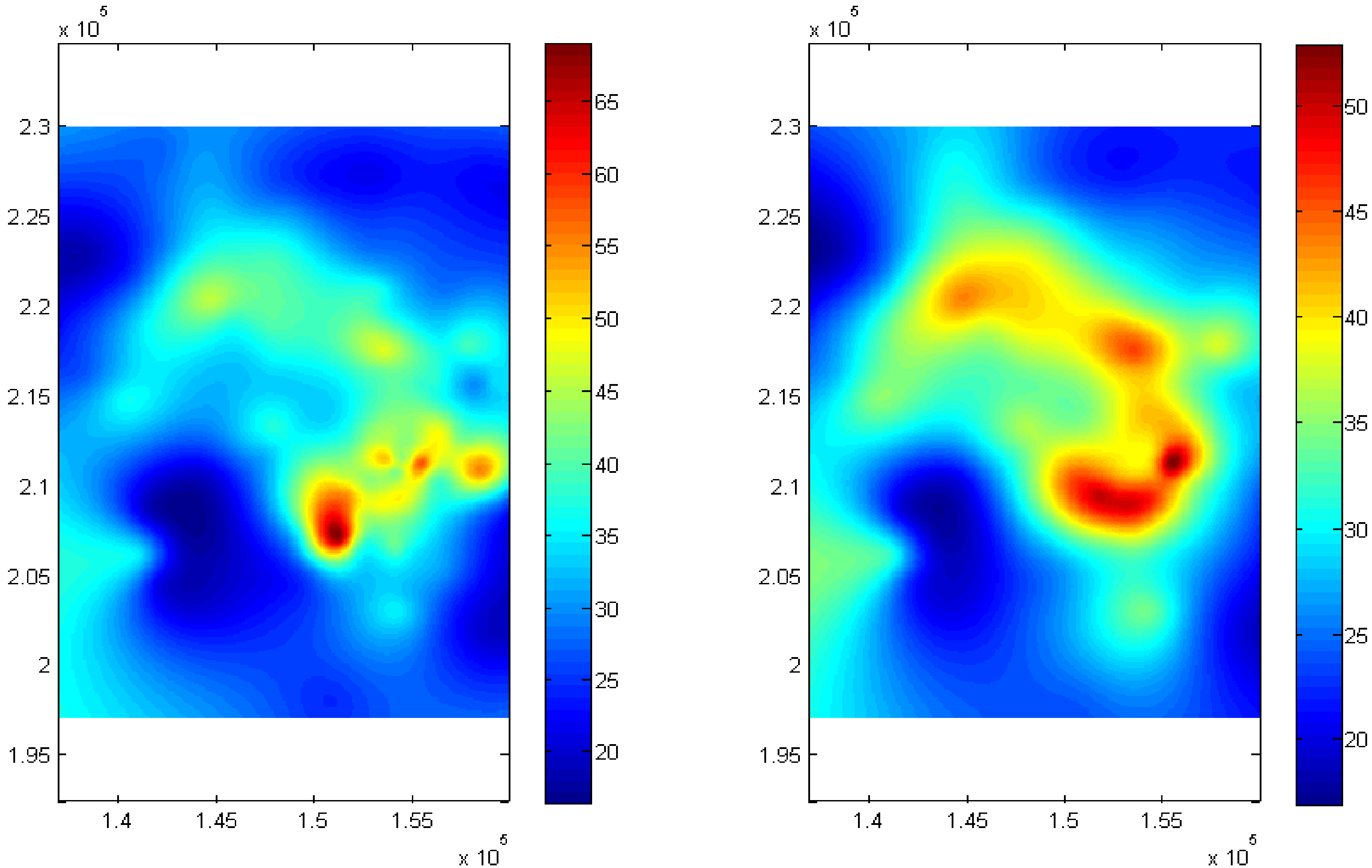
Basic test case

Application – Test case N.2: Antwerp



Impact of the station uncertainty of the stations

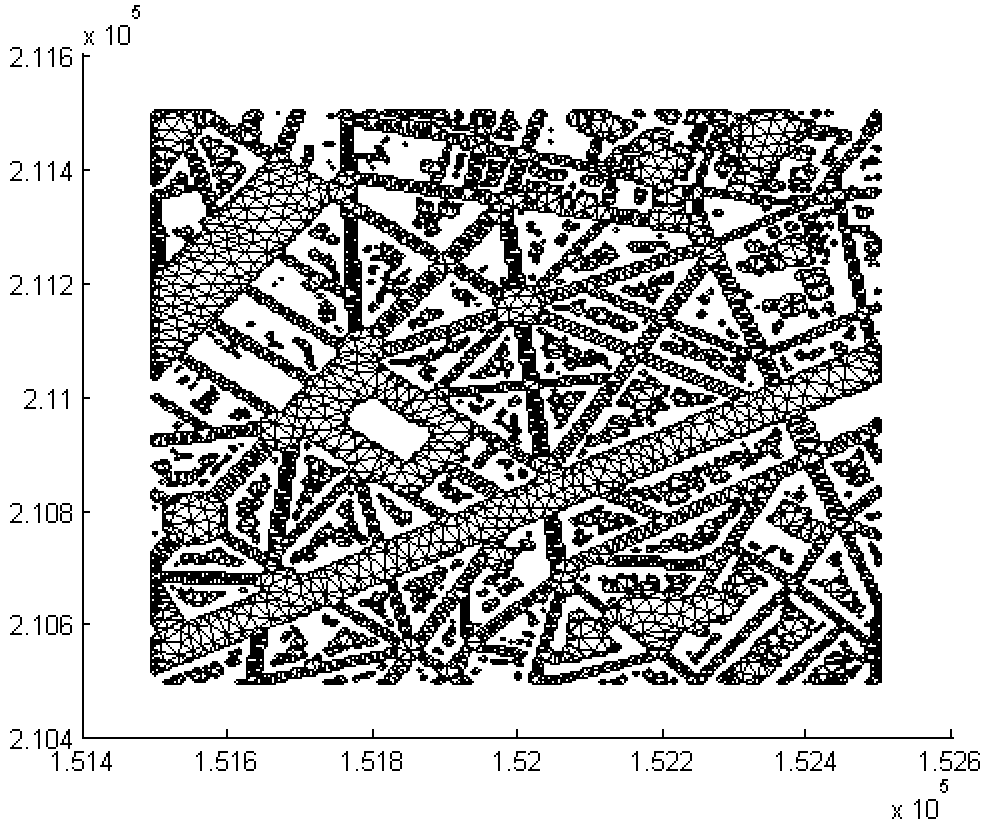
Application – Test case N.2: Antwerp



Impact of the non-street-canyon stations

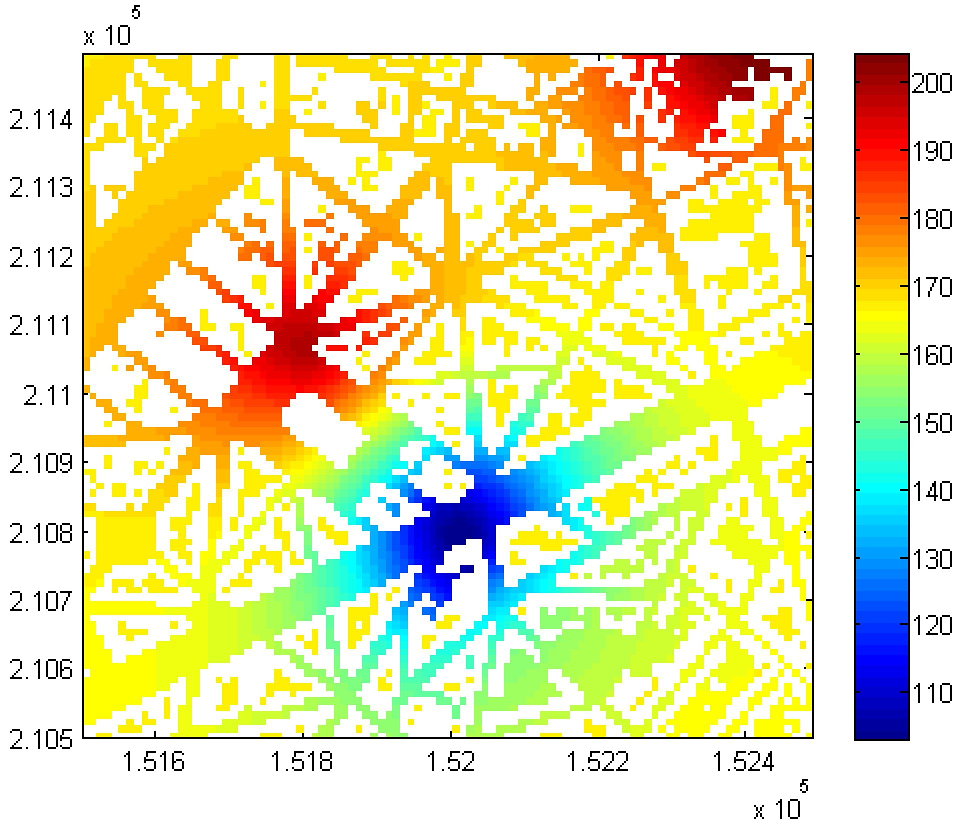
Application – Test case N.2: Antwerp

Physical constraint:
Information propagation limited to the domain of interest



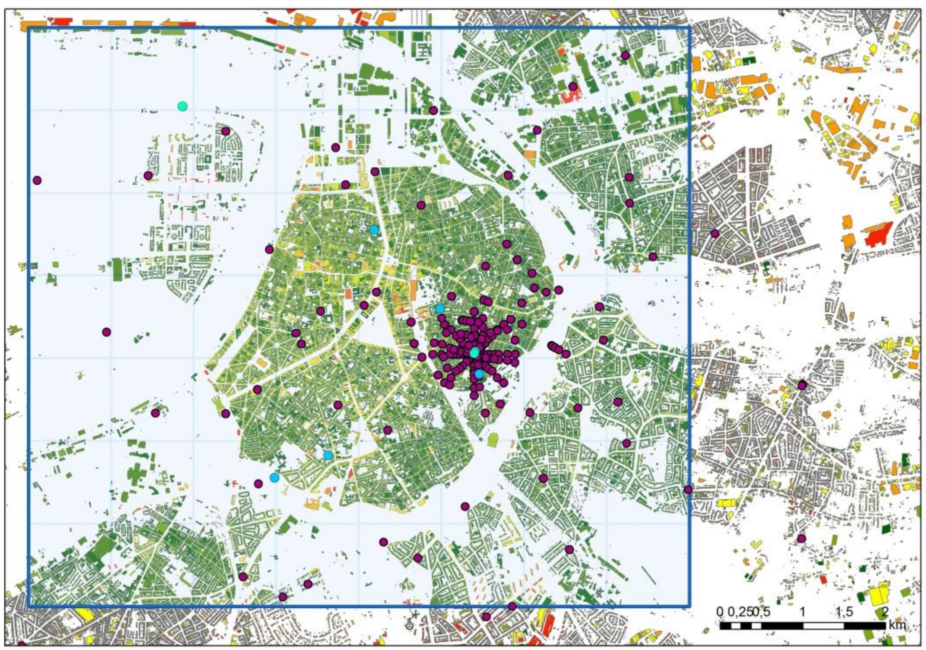
Application – Test case N.2: Antwerp

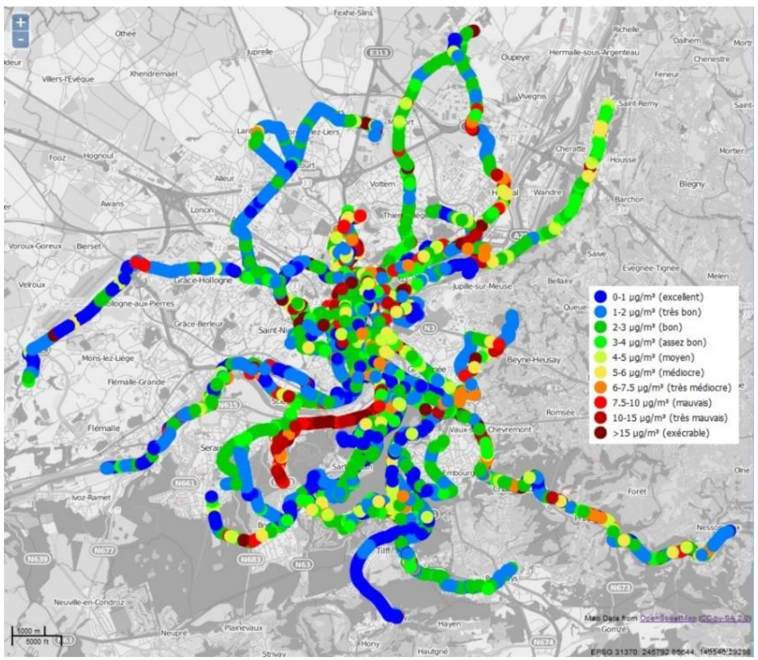
Results for a three-point test case



Perspectives

- Generalization of the method to merge results from tiles
- Quantitative evaluation of the method's benefit
- Use of the temporal detrending
- Use of physical constraints
- Extensive use of the method to itinerary campaigns





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[2] Bretherton et al. (1976). A technique for objective analysis and design of oceanographic instruments applied to MODE-73. Deep-Sea Research, 23: 559–582. doi:10.1016/00117471(76)90001-2.

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[9] http://modb.oce.ulg.ac.be/mediawiki/index.php/DIVA

Acknowledgments

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ISSeP
Institut scientifique de service public
Métrologie environnementale
Recherche - Analyses
Essais - Expertises

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